Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 18, 2003*
Revised: *July 1, 2003*
Accepted: *July 2, 2003*

Odd-intrinsic-parity processes within the resonance effective theory of QCD

Pedro D. Ruiz-Femenía, A. Pich and J. Portolés

Departament de Física Teòrica, IFIC, Universitat de València - CSIC
Apt. Correus 22085, E-46071 València, Spain
E-mail: pedro.ruiz@ific.uv.es, antonio.pich@uv.es, jorge.portoles@ific.uv.es

Abstract: We analyse the most general odd-intrinsic-parity effective lagrangian of QCD valid for processes involving one pseudoscalar with vector mesons described in terms of antisymmetric tensor fields. Substantial information on the odd-intrinsic-parity couplings is obtained by constructing the vector-vector-pseudoscalar Green's three-point function, at leading order in $1/N_C$, and demanding that its short-distance behaviour matches the corresponding OPE result. The QCD constraints thus enforced allow us to predict the decay amplitudes $\omega \to \pi\gamma$ and $\rho \to \pi\gamma$, and the $\mathcal{O}(p^6)$ corrections to $\pi \to \gamma\gamma$. Noteworthy consequences concerning the vector meson dominance assumption in the decay $\omega \to 3\pi$ are also extracted from the previous analysis.

Keywords: 1/N Expansion, Global Symmetries, QCD, Chiral Lagrangians.

JHEP07(2003)003

Contents

1. Introduction

Effective field theories of QCD have provided efficient ways to explore hadron dynamics in those regimes where we are not able to solve the full theory. Built from the same principles and symmetries which govern QCD, the effective actions put at our disposal a model-independent framework to generate the interactions between the active degrees of freedom. In the very low-energy domain, chiral perturbation theory [1, 2, 3] has achieved a remarkable success in describing the strong interactions among pseudoscalar mesons. Moving up to the 1 GeV region has been proved more difficult, as the effects of vector resonances become dominant and must be accommodated in the theory. Several works [2, 4, 5] have provided a sound procedure to include resonance states within the chiral framework, later christened Resonance Chiral Theory. This approach, however, leaves the couplings entering the effective lagrangian unknown, as they are not fixed by the symmetry alone. One should then rely on the phenomenology or, alternatively, construct theoretical tools that could provide a meaningful way to compare the results of the effective theory with those of QCD. The pioneering work of ref. [6] indicated that the analysis of Green's functions and form factors of QCD currents yields valuable information on the resonance sector and, at the same time, clarifies the ambiguities related to the choice of the Lorentz group representation for the resonance fields.

Recently, several authors have pushed forward this direction, either by using a lagrangian with explicit resonance degrees of freedom [7], or within the framework of the lowest meson dominance (LMD) approximation to the large number of colours (N_C) limit of QCD [7]–[11]. In particular, the authors of ref. [7] undertook a systematic study of several QCD three-point functions which share the property of being zero in absence of spontaneous chiral symmetry breaking for massless quarks. This common feature means that these Green's functions are free of perturbative contributions from QCD at short distances. Therefore, their OPE expansion, although formally applicable in the high-energy region, should be more reliable when descending to energies close to the resonance region, thus supporting the idea that a smooth matching between QCD and the effective description involving resonances may exist for these functions. Under this hypothesis, it was shown in ref. [7] that while the ansatz derived from the LMD approach automatically incorporates the right short-distance behaviour of QCD by construction, the same Green's functions as calculated with a resonance lagrangian, in the vector-field representation, are incompatible with the OPE outcome. Thus, the $\mathcal{O}(p^6)$ low-energy constants they extract from the resonance lagrangian differ from the estimates of the LMD ansatz. Moreover the authors put forward that these discrepancies cannot be repaired just by introducing local counterterms from the chiral lagrangian $\mathcal{L}_\chi^{(6)}$, as it was done at $\mathcal{O}(p^4)$ in ref. [6]. New terms with resonance fields and higher-order derivatives need to be added, at least in the vector-field representation, but the general procedure remains unknown.

The result above severely questions the usefulness of the resonance effective theory beyond the initial work of ref. [6], that relies not only on the QCD global symmetries but also on the fact that its large-N_C limit resembles, at least qualitatively, the three colour theory [12, 13]. In addition one of the basic tenets, after the conclusions of ref. [13], is that meson physics in the large-N_C limit is described by the tree diagrams of an effective local lagrangian, with local vertices and local meson fields. Hence after the qualm put forward by ref. [7] we think that this issue deserves further investigation. With this aim, we have reanalysed one of the Green's function studied in this last reference, the vector-vector-pseudoscalar three-point function, this time with the vector mesons described in terms of antisymmetric tensor fields.

The latter study requires the introduction of an odd-intrinsic-parity effective lagrangian in the formulation of ref. [5] containing all allowed interactions between two vector objects (currents or resonances) and one pseudoscalar meson. After a brief introduction on chiral theory, section 2 of this paper is devoted to this subject. In section 3 we evaluate the vector-vector-pseudoscalar three-point function $\langle$VVP$\rangle$ within our effective theory at leading order in the $1/N_C$ expansion. We recall its short-distance properties, as obtained from the OPE calculation, and then we demand that the $\langle$VVP$\rangle$ Green's function built with the effective action with unknown parameters matches the same behaviour. The set of relations among couplings derived is then tested in several intrinsic-parity-violating decays in section 4. Finally, we give our conclusions.

2. Resonance Chiral Theory and the odd-intrinsic-parity sector

The low-energy behaviour of QCD for the light quark sector (u, d, s) is known to be ruled by the spontaneous breaking of chiral symmetry giving rise to the lightest hadron degrees of freedom, identified with the octet of pseudoscalar mesons. The corresponding effective

JHEP07(2003)003

realization of QCD describing the interaction between the Goldstone fields is called chiral perturbation theory [1, 2, 3]. The effective lagrangian to lowest order in derivatives, $\mathcal{O}(p^2)$, is given by:

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \,, \tag{2.1}$$

where

$$\begin{aligned} u_\mu &= i \left[u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i \ell_\mu) u^\dagger \right] , \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,, \qquad \chi = 2B_0 (s + ip) \,. \end{aligned} \tag{2.2}$$

The unitary matrix in flavour space

$$u(\phi) = \exp \left\{ i \frac{\Phi}{\sqrt{2}\, F} \right\} , \tag{2.3}$$

is a (non-linear) parameterization of the Goldstone octet of fields:

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}} \, \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} . \tag{2.4}$$

The external hermitian matrix fields r_μ, ℓ_μ, s and p promote the global $\mathrm{SU(3)_R \times SU(3)_L}$ symmetry of the lagrangian to a local one, and generate Green functions of quark currents by taking appropriate functional derivatives. Interactions with electroweak bosons can be accommodated through the vector $v_\mu = (r_\mu + \ell_\mu)/2$ and axial-vector $a_\mu = (r_\mu - \ell_\mu)/2$ fields, while the scalar field s provides a very convenient way of incorporating explicit chiral symmetry breaking through the quark masses

$$s = \mathcal{M} + \cdots \,, \qquad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s) \,.$$

The generating functional $Z[v, a, s, p]$ calculated in terms of the external sources is manifestly chiral invariant, but the physically interesting Green functions (with broken chiral symmetry) are obtained by taking a particular direction in flavour space through functional differentiation. Finally, the $\mathcal{L}_\chi^{(2)}$ lagrangian is settled by fixing the unknown F and B_0 parameters from the phenomenology: $F \simeq F_\pi \simeq 92.4\,\mathrm{MeV}$ is the decay constant of the charged pion and $B_0 F^2 = -\langle 0 | \bar{\psi}\psi | 0 \rangle_0$ in the chiral limit.

Spectroscopy reveals the existence of vector meson resonances as we approach the 1 GeV energy region. These can be classified in $\mathrm{SU(3)_V}$ octets and must be included as explicit degrees of freedom in order to describe hadron dynamics [4]. At the lowest order in derivatives, the chiral invariant lagrangian for the vector mesons and their interaction with Goldstone fields reads [5], in the antisymmetric tensor formulation,

$$\mathcal{L}_\mathrm{V} = \mathcal{L}_\mathrm{Kin}(V) + \mathcal{L}_2(V) \,, \tag{2.5}$$

with kinetic terms

$$\mathcal{L}_\mathrm{Kin}(V) = -\frac{1}{2} \left\langle \nabla^\lambda V_{\lambda\mu} \nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2} V_{\mu\nu} V^{\mu\nu} \right\rangle , \tag{2.6}$$

JHEP07(2003)003

where M_V is the mass of the lowest octet of vector resonances under $\mathrm{SU}(3)_\mathrm{V}$, and the covariant derivative

$$\nabla_\mu V = \partial_\mu V + [\Gamma_\mu, V]\,, \qquad \Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\}\,,$$

is defined in such a way that $\nabla_\mu V$ also transforms as an octet under the action of the group. For the interaction lagrangian $\mathcal{L}_2(V)$ we have

$$\mathcal{L}_2(V) = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle\,, \tag{2.7}$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u\,, \tag{2.8}$$

with $F^{\mu\nu}_{L,R}$ the field strength tensors of the left and right external sources ℓ_μ and r_μ, and F_V, G_V are real couplings. The octet fields are written in the usual matrix notation

$$V_{\mu\nu} = \frac{\vec{\lambda}}{\sqrt{2}}\vec{V}_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 \end{pmatrix}_{\mu\nu}\,. \tag{2.9}$$

The chiral couplings contained in $\mathcal{L}_2(V)$ only concern the even-intrinsic-parity sector. In ref. [6] it was shown that, up to $\mathcal{O}(p^4)$ in the chiral counting, the effective lagrangian $\mathcal{L}_{\chi V} \equiv \mathcal{L}_\chi^{(2)} + \mathcal{L}_\mathrm{V}$ is enough to satisfy the short-distance QCD constraints where vector resonances play a significant role. For the odd-intrinsic-parity sector, three different sources might be considered: (i) the Wess-Zumino action [14], which is $\mathcal{O}(p^4)$ and fulfills the chiral anomaly, (ii) chiral invariant $\epsilon_{\mu\nu\rho\sigma}$ terms involving vector mesons which, upon integration, will start to contribute at $\mathcal{O}(p^6)$ in the antisymmetric formulation, and (iii) the relevant operators in the $\mathcal{O}(p^6)$ Goldstone chiral lagrangian [15, 16]. All of them may contribute to the $\langle$VVP$\rangle$ Green's function.

The chiral anomaly is driven by the Wess-Zumino action $Z_\mathrm{WZ}[v,a]$. We do not recall its functional here and address the reader to ref. [17] for the explicit expression. On the other side effective odd-intrinsic-parity lagrangians with vector resonances have been previously considered in the literature in order to study the equivalence of different vector resonance models to reproduce the one-loop divergences of the Wess-Zumino action [18], in the context of the extended Nambu-Jona-Lasinio model [19], or to estimate the low-energy constants of the $\mathcal{O}(p^6)$ Goldstone chiral lagrangian [7]. Within the antisymmetric formalism, we shall build an independent set of odd-intrinsic-parity operators which comprise all possible vertices involving two vector resonances and one pseudoscalar (VVP), and vertices with one vector resonance and one external vector source plus one pseudoscalar (VJP).

The building blocks for these terms are the ones defined above, which share the right properties under chiral transformations. Besides, the terms must satisfy Lorentz, P and C invariance. Other useful relations to reduce the number of independent terms and construct the basis are detailed in the appendix. Our basis reads:[1]

[1] We use the convention $\epsilon_{0123} = +1$ for the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ throughout this paper

JHEP07(2003)003

VJP terms

$$
\begin{aligned}
\mathcal{O}^1_{\rm VJP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ V^{\mu\nu}, f_+^{\rho\alpha} \} \nabla_\alpha u^\sigma \, \rangle \, , \\
\mathcal{O}^2_{\rm VJP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ V^{\mu\alpha}, f_+^{\rho\sigma} \} \nabla_\alpha u^\nu \, \rangle \, , \\
\mathcal{O}^3_{\rm VJP} &= i \, \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ V^{\mu\nu}, f_+^{\rho\sigma} \} \, \chi_- \, \rangle \, , \\
\mathcal{O}^4_{\rm VJP} &= i \, \epsilon_{\mu\nu\rho\sigma} \, \langle \, V^{\mu\nu} \, [\, f_-^{\rho\sigma}, \chi_+] \, \rangle \, , \\
\mathcal{O}^5_{\rm VJP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ \nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha} \} u^\sigma \, \rangle \, , \\
\mathcal{O}^6_{\rm VJP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ \nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma} \} u^\nu \, \rangle \, , \\
\mathcal{O}^7_{\rm VJP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ \nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha} \} u_\alpha \, \rangle \, ,
\end{aligned}
\tag{2.10}
$$

VVP terms

$$
\begin{aligned}
\mathcal{O}^1_{\rm VVP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ V^{\mu\nu}, V^{\rho\alpha} \} \nabla_\alpha u^\sigma \, \rangle \, , \\
\mathcal{O}^2_{\rm VVP} &= i \, \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ V^{\mu\nu}, V^{\rho\sigma} \} \, \chi_- \, \rangle \, , \\
\mathcal{O}^3_{\rm VVP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ \nabla_\alpha V^{\mu\nu}, V^{\rho\alpha} \} u^\sigma \, \rangle \, , \\
\mathcal{O}^4_{\rm VVP} &= \epsilon_{\mu\nu\rho\sigma} \, \langle \, \{ \nabla^\sigma V^{\mu\nu}, V^{\rho\alpha} \} u_\alpha \, \rangle \, .
\end{aligned}
\tag{2.11}
$$

The operators with $\chi_\pm$ break $\mathrm{SU(3)_V}$ symmetry when distinct quark masses are introduced through the external scalar field $s = \mathcal{M} + \cdots$. However, only the pseudoscalar source p in $\mathcal{O}^3_{\rm VJP}$ and $\mathcal{O}^2_{\rm VVP}$ will enter our calculation of the Green's function, while $\mathcal{O}^4_{\rm VJP}$ will not contribute at all and has just been included in the VJP basis for completeness.

The authors of ref. [18] also built the VVP operators in the tensor-field representation and further constrained the number of independent operators to three by applying the equation of motion of the pseudoscalar field at lowest order; some care is needed in our case, as particles inside Green's functions are not on their mass shell. The resonance lagrangian for the odd-intrinsic-parity sector will thus be defined as

$$
\begin{aligned}
\mathcal{L}_V^{\rm odd} &= \mathcal{L}_{\rm VJP} + \mathcal{L}_{\rm VVP} \, , \\
\mathcal{L}_{\rm VJP} &= \sum_{a=1}^{7} \frac{c_a}{M_V} \, \mathcal{O}^a_{\rm VJP} \, , \qquad \mathcal{L}_{\rm VVP} = \sum_{a=1}^{4} d_a \, \mathcal{O}^a_{\rm VVP} \, .
\end{aligned}
\tag{2.12}
$$

The octet mass M_V has been introduced in $\mathcal{L}_{\rm VJP}$ to define dimensionless c_a couplings. We stress that the set defined above is a complete basis for constructing vertices with only one-pseudoscalar; for a larger number of pseudoscalars additional operators may emerge.

Finally we have to pay attention to the $\mathcal{O}(p^6)$ Goldstone chiral lagrangian. Two operators may contribute at leading order in the $1/N_C$ expansion to the $\langle \mathrm{VVP} \rangle$ Green's function:

$$
\mathcal{L}^{(6)}_{\rm odd} = i \, \epsilon_{\mu\nu\alpha\beta} \left\{ t_1 \, \langle \, \chi_- f_+^{\mu\nu} f_+^{\alpha\beta} \, \rangle - i \, t_2 \, \langle \nabla_\lambda \, f_+^{\lambda\mu} \{ f_+^{\alpha\beta}, u^\nu \} \, \rangle \right\} . \tag{2.13}
$$

The t_i couplings are in principle unknown.[2] These operators belong both to the effective theory where resonances are still active degrees of freedom and to the theory where those

[2]In the notation of ref. [16] $t_1 = C_7^W$ and $t_2 = -C_{22}^W$.

JHEP07(2003)003

have been integrated out. Hence in the latter case the couplings can be split as $t_i = t_i^R + \hat{t}_i$ where t_i^R is generated by the integration of resonances and $\hat{t}_i$ is a remainder that may survive in the effective theory where resonances are still active. Vector and pseudoscalar resonances can contribute, in principle, to t_1^R, though the latter are suppressed because of their higher masses. Therefore we will consider that $t_1^R \simeq t_1^V$. Meanwhile t_2 has only vector resonance contributions and then $t_2^R = t_2^V$. Indeed by integrating out the vector mesons in $\mathcal{L}_{\rm V} + \mathcal{L}_{\rm V}^{\rm odd}$ we obtain:

$$\begin{aligned} t_1^V &= -\frac{F_V}{4\sqrt{2}M_V^3}\left[c_1 + c_2 + 8c_3 - c_5\right] + \frac{F_V^2}{8M_V^4}\left[d_1 + 8d_2 - d_3\right], \\ t_2^V &= -\frac{F_V}{\sqrt{2}M_V^3}\left(c_5 - c_6\right) + \frac{F_V^2}{2M_V^4}\, d_3\,. \end{aligned} \tag{2.14}$$

On the other side the successful resonance saturation of the chiral lagrangian couplings at $\mathcal{O}(p^4)$ [5] might translate naturally to $\mathcal{O}(p^6)$ couplings too, implying that $\hat{t}_i$ could be neglected. We will attach to this point and will assume that the t_i couplings are generated completely through integration of vector resonances. Accordingly we should not include $\mathcal{L}_{\rm odd}^{(6)}$ in our evaluation of the Green's function in order not to double count degrees of freedom. We shall come back to this discussion in the next section.

In summary we will proceed in the following by considering the relevant effective resonance theory (ERT) given by:

$$Z_{\rm ERT}[v,a,s,p] = Z_{\rm WZ}[v,a] + Z_{{\rm V}\chi}^{\rm odd}[v,a,s,p]\,, \tag{2.15}$$

where $Z_{{\rm V}\chi}^{\rm odd}[v,a,s,p]$ is generated by $\mathcal{L}_\chi^{(2)}$ in eq. (2.1), $\mathcal{L}_{\rm V}$ in eq. (2.5) and $\mathcal{L}_V^{\rm odd}$ in eq. (2.12).

3. Short-distance information on the odd-intrinsic-parity couplings

The construction of an effective field theory that satisfies the symmetry requirements of QCD is a model-independent procedure to accomplish the low-energy properties of the theory without missing essential dynamics. The price to pay for the universality of such approach is an increasing number of (a priori) unknown low-energy constants as we tend to improve the accuracy of our calculations, which eventually reflects in a loss of predictive power. Comparison with data has been a fruitful way to extract the values of most of the chiral couplings up to $\mathcal{O}(p^4)$, as well as some of the resonance parameters for the lightest vector octet and, to a small extent, for the axial-vector, scalar and pseudoscalar resonances.

Jointly with the experimental determination, alternative ways to infer the values of the resonance couplings have been explored. Thus the QCD ruled short-distance behaviour of the vector and axial form factors in the large-N_C limit (approximated with only one octet of vector resonances) constrains the couplings of $\mathcal{L}_2(V)$ in eq. (2.7), that must satisfy [6]:

$$1 - \frac{F_V\, G_V}{F^2} = 0\,, \tag{3.1}$$

$$2F_V\, G_V - F_V^2 = 0\,, \tag{3.2}$$

and predict $F_V = \sqrt{2}\,F$ and $G_V = F/\sqrt{2}$, in excellent agreement with the phenomenology. The strict large-N_C limit would demand that the full spectrum of infinite zero-width vector resonances should be included in the evaluation of the form factors above. However the agreement with data suggests that the approximation of the lightest vector multiplet resembles the limit. This is the basic assumption of the LMD approach.

In addition, the study of the short-distance properties of Green's functions and the comparison with the same objects built from the effective action with explicit resonance degrees of freedom can yield relevant information on the resonance couplings, as explored in previous works [7]–[11]. We now follow this method to impose restrictions on the new couplings introduced in the odd-intrinsic-parity sector.

The relevant Green's function for this purpose is the vector-vector-pseudoscalar QCD three-point function $\langle \mathrm{VVP} \rangle$,

$$(\Pi_{\mathrm{VVP}})^{(abc)}_{\mu\nu}(p,q) = \int d^4x \int d^4y \, e^{i(p\cdot x + q \cdot y)} \langle 0 | \, T \left[V^a_\mu(x) \, V^b_\nu(y) \, P^c(0) \right] |0\rangle \,, \tag{3.3}$$

which requires the octet vector current,

$$V^a_\mu(x) = \left(\bar{\psi} \gamma_\mu \frac{\lambda^a}{2} \psi \right)(x) \,, \tag{3.4}$$

and the octet pseudoscalar density

$$P^a(x) = \left(\bar{\psi} i \gamma_5 \frac{\lambda^a}{2} \psi \right)(x) \,. \tag{3.5}$$

The invariances of QCD under parity and time-reversal transformations allow us to extract the group and tensor structure of $\langle \mathrm{VVP} \rangle$ in the $\mathrm{SU(3)_V}$ limit,

$$(\Pi_{\mathrm{VVP}})^{(abc)}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} \, p^\alpha q^\beta d^{abc} \, \Pi_{\mathrm{VVP}} \left(p^2, q^2, r^2 \right) \,, \tag{3.6}$$

with the four-vector $r = -(p+q)$. The first situation concerning the short-distance behaviour of the $\langle \mathrm{VVP} \rangle$ Green's function that we can analyse is the case when both momenta p, q in Π_{VVP} become simultaneously large. The QCD calculation within the OPE framework gives, in the chiral limit and up to corrections of $\mathcal{O}(\alpha_s)$, [9]:

$$\lim_{\lambda \to \infty} \Pi_{\mathrm{VVP}}((\lambda p)^2, (\lambda q)^2, (\lambda p + \lambda q)^2) = - \frac{\langle \bar{\psi}\psi \rangle_0}{2\lambda^4} \frac{p^2+q^2+r^2}{p^2 q^2 r^2} + \mathcal{O}\left(\frac{1}{\lambda^6} \right) \,, \tag{3.7}$$

where $\langle \bar{\psi}\psi \rangle_0$ is the single flavour bilinear quark condensate. $(\Pi_{\mathrm{VVP}})^{(abc)}_{\mu\nu}$ is an order parameter of the spontaneous breaking of chiral symmetry. Hence, in the chiral limit, it does not receive contributions from perturbative QCD at large momentum transfers. This non-perturbative feature is in fact desirable to guarantee that the OPE domain of applicability can be enlarged down to the 1-2 GeV energy region.

In position space, eq. (3.7) corresponds to the limit where the space-time arguments of the three operators in $\langle \mathrm{VVP} \rangle$ approach the same point at equal rates. We can also demand that only the argument of two of the three operators converge towards the same point [7]. In this case two situations arise: either the two vector currents are taken at the same point, or one of the vector currents and the pseudoscalar density are evaluated at the same argument. The first situation was exploited in the analysis of the decay of pseudoscalars into lepton pairs of ref. [11].

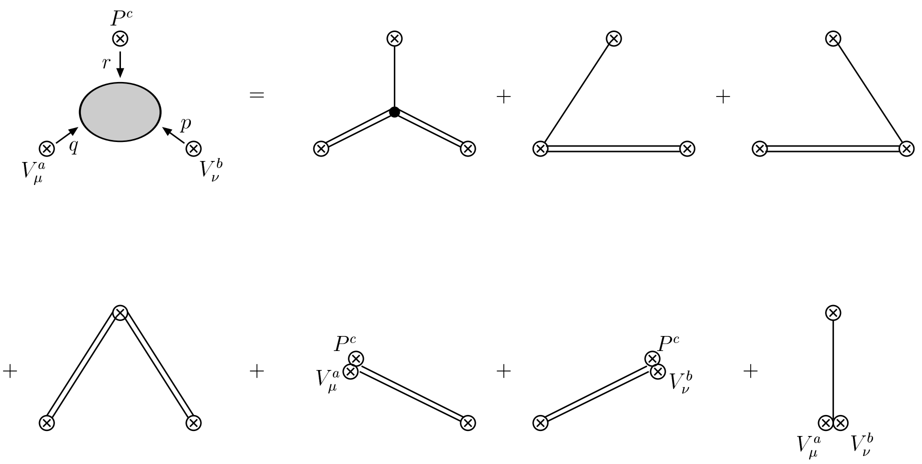


Figure 1: Diagrams entering the calculation of the VVP 3-point function with the ERT action. Double lines represent vector resonances, single lines are short for pseudoscalar mesons.

We shall now build the $\langle \mathrm{VVP} \rangle$ Green's function with the effective resonance theory given by $Z_{\mathrm{ERT}}[v, a, s, p]$, and impose that the short-distance constraint in eq. (3.7) is fulfilled.

At leading order in the $1/N_C$ expansion of QCD, the three-point correlator is evaluated from the tree-level diagrams shown in figure 1. In this limit, an infinite spectrum of zero-width vector resonances should be considered in each channel. Fortunately, the LMD approximation to large-N_C, which assumes that a single resonance in each channel saturates the requirements of QCD, can be invoked as a first test of the short-distance behaviour of our Green's function. Indeed, we shall prove that this approximation is sufficient to satisfy the short-distance QCD constraints commented above.

The couplings of the resonances among themselves and to the external sources have been detailed in eq. (2.15), and the chiral limit is implied throughout. Our result reads

$$
\begin{aligned}
\Pi^{\mathrm{res}}_{\mathrm{VVP}}(p^2, q^2, r^2) = &-\frac{\langle \bar{\psi}\psi \rangle_0}{F^2} \times \\
&\times \Bigg\{ 4\, F_V^2\, \frac{(d_1 - d_3)\, r^2 + d_3 (p^2 + q^2)}{(M_V^2 - p^2)\,(M_V^2 - q^2)\, r^2} - \\
&\quad -2\sqrt{2}\, \frac{F_V}{M_V} \frac{r^2(c_1+c_2-c_5)+p^2(-c_1+c_2+c_5-2c_6)+q^2(c_1-c_2+c_5)}{(M_V^2 - p^2)\, r^2} - \\
&\quad -2\sqrt{2}\, \frac{F_V}{M_V} \frac{r^2(c_1+c_2-c_5)+q^2(-c_1+c_2+c_5-2c_6)+p^2(c_1-c_2+c_5)}{(M_V^2 - q^2)\, r^2} + \\
&\quad + \frac{32 F_V^2\, d_2}{(M_V^2-p^2)(M_V^2-q^2)} - \frac{16\sqrt{2} F_V c_3}{M_V (M_V^2-p^2)} - \frac{16\sqrt{2} F_V c_3}{M_V (M_V^2-q^2)} - \frac{N_C}{8\pi^2 r^2} \Bigg\} .
\end{aligned}
\tag{3.8}
$$

The contributions in eq. (3.8) have been written following the same ordering of figure 1 (left to right, top to bottom). The last term originates from the piece of the Wess-Zumino

JHEP07(2003)003

action $Z_{\rm WZ}[v,a]$ responsible of the pseudoscalar meson decays into two photons,

$$\mathcal{L}_{\rm WZ}^{(4)} = - \frac{\sqrt{2}\, N_C}{8\pi^2\, F}\; \epsilon_{\mu\nu\alpha\beta}\, \langle \Phi\, \partial^\mu v^\nu\, \partial^\alpha v^\beta \,\rangle \, . \tag{3.9}$$

If we now take the limit of two momenta becoming simultaneously large in $\Pi_{\rm VVP}^{\rm res}$, we find compatibility with the QCD short-distance constraint up to order $1/\lambda^4$, eq. (3.7), provided the following conditions among the $\mathcal{L}_V^{\rm odd}$ couplings hold:

$$\begin{aligned} 4\,c_3 + c_1 &= 0\,, \\ c_1 - c_2 + c_5 &= 0\,, \\ c_5 - c_6 &= \frac{N_C}{64\pi^2}\frac{M_V}{\sqrt{2}F_V}\,, \\ d_1 + 8\,d_2 &= -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} + \frac{F^2}{4F_V^2}\,, \\ d_3 &= -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} + \frac{F^2}{8F_V^2}\,. \end{aligned} \tag{3.10}$$

These relations have been obtained within the chiral limit. However the couplings of the Effective lagrangian do not depend on the masses of the Goldstone fields and, consequently, the constraints in eq. (3.10) apply for non-zero pseudoscalar masses too.

Actually our $\langle$VVP$\rangle$ three-point function fully reproduces the LMD ansatz suggested in ref. [9]:

$$\Pi_{\rm VVP}^{\rm res}(p^2,q^2,(p+q)^2) = -\frac{\langle\bar{\psi}\psi\rangle_0}{2}\cdot\frac{(p^2+q^2+r^2) - \frac{N_C}{4\pi^2}\frac{M_V^4}{F^2}}{(p^2-M_V^2)(q^2-M_V^2)r^2}\,. \tag{3.11}$$

As a consequence both the short-distance behaviour in eq. (3.7) and those conditions where two vector currents or one vector current and the pseudoscalar density meet at the same point, mentioned above, are thoroughly satisfied.

The ansatz (3.11) implies that we recover the LMD estimates for the low-energy constants derived in ref. [7]. The authors of this reference found that the same agreement with the short and long-distance QCD behaviour could not be reached working with the resonance lagrangian in the vector representation, not even at the expense of introducing local contributions from the $\mathcal{O}(p^6)$ chiral lagrangian. They then suggested that the problem may be inherent to the effective lagrangian approach and unlikely to be fixed just by using other representations for the resonance fields; our result, derived in the antisymmetric tensor-field formulation with an odd-intrinsic-parity sector, contradicts this assertion, at least in what concerns the $\langle$VVP$\rangle$ Green's function.

Finally it is worth to comment the situation that would arise if local $\mathcal{O}(p^6)$ operators of the chiral lagrangian in eq. (2.13) were introduced in this analysis. We argued in section 2 that the couplings of those operators, t_i, could be completely saturated by vector resonances and, accordingly, $t_i \simeq t_i^V$ and $\hat{t}_i \simeq 0$. If we include a non-vanishing $\hat{t}_i$ in the evaluation of the Green's function carried above it is easy to see that the high energy behaviour is spoiled unless higher-derivative couplings with resonances are considered. If we stay within our $Z_{\rm ERT}[v,a,s,p]$ action, that satisfies by itself the matching with the QCD result, the OPE imposes $\hat{t}_i = 0$, $i = 1,2$.

It is also interesting to notice that the combinations of odd-intrinsic couplings which appear in the expressions of t_i^V, eq. (2.14) are predicted from the QCD conditions above. We obtain:

$$
\begin{aligned}
t_1^{\mathrm{V}} &= \frac{F^2}{64\, M_V^4}\,, \\
t_2^{\mathrm{V}} &= -\frac{N_C}{64\,\pi^2\, M_V^2}\left[1 - \frac{4\,\pi^2}{N_C}\,\frac{F^2}{M_V^2}\right],
\end{aligned}
\tag{3.12}
$$

which coincide[3] with the predictions made for these parameters in [9]. This fact is not surprising, since the relations (3.12) were derived in [9] by expanding the $\langle\mathrm{VVP}\rangle$ ansatz, eq. (3.11), at low-momenta and comparing it with the $\langle\mathrm{VVP}\rangle$ expression obtained from the $\mathcal{L}_{\mathrm{odd}}^{(6)}$ lagrangian. The success in reproducing the same representation for Π_{VVP} within the resonance effective lagrangian has automatically generated identical values for the chiral parameters.[4]

4. Phenomenology of intrinsic-parity violating processes

Odd-intrinsic-parity processes have been widely studied within chiral perturbation theory where resonances are integrated out [20]. In order to gain more insight on the odd-intrinsic-parity sector of the resonance lagrangian and, to make some test on the validity of the short-distance conditions obtained above, we study in this section the processes $\omega \to \pi\gamma$, $\rho \to \pi\gamma$, $\omega \to 3\pi$ and $\pi \to 2\gamma$.

4.1 $\omega \to \pi\gamma$

At tree-level, the intrinsic-parity violating transition $\omega \to \pi\gamma$ receives contributions from both the VJP and VVP terms of $\mathcal{L}_V^{\mathrm{odd}}$. The corresponding diagrams are displayed in figure 2. The physical ω resonance is a superposition of an octet component, ω_8, and a singlet one, ω_1, which can be added as a diagonal matrix $\omega_1/\sqrt{3}$ to the octet, eq. (2.9); if ideal mixing is assumed then the states of defined mass are

$$
|\,\omega\,\rangle = \sqrt{\frac{2}{3}}\,|\,\omega_1\rangle + \sqrt{\frac{1}{3}}\,|\,\omega_8\rangle\,,
$$

and

$$
|\,\phi\,\rangle = -\sqrt{\frac{1}{3}}\,|\,\omega_1\rangle + \sqrt{\frac{2}{3}}\,|\,\omega_8\rangle\,.
$$

[3]There is a minus sign difference in the definitions of t_1 and t_2 in [9] because the convention used there for the Levi-Civita tensor is the opposite to ours.

[4]The author of [9] extended the results for t_1^V and t_2^V above by including an additional pole-contribution in the VVP ansatz from a pseudoscalar $\pi(1300)$ resonance.

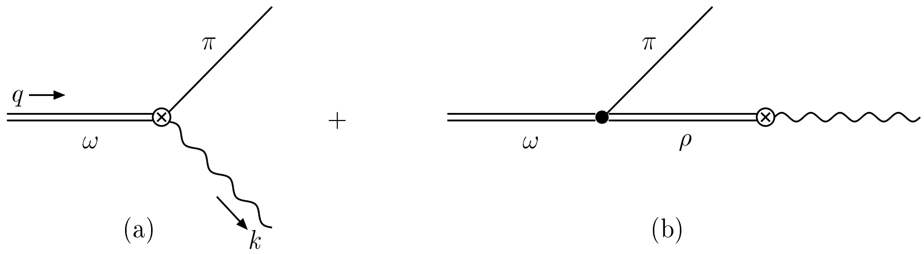


Figure 2: Lowest order diagrams for the process $\omega \to \pi\gamma$.

The amplitudes for the direct and ρ-mediated diagrams, figures 2a and 2b respectively, read

$$\begin{aligned} i\,\mathcal{M}^{direct}_{\omega\to\pi\gamma} &= i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon^{\alpha}_{\omega}\epsilon^{\beta}_{\gamma} q^{\rho}k^{\sigma}\,2\sqrt{2}\,\frac{e}{M_\omega M_V F}\times \\ &\quad\times\Big[(c_2-c_1+c_5-2c_6)\,M_\omega^2+(c_1+c_2+8c_3-c_5)\,m_\pi^2\Big]\,, \\ i\,\mathcal{M}^{\rho}_{\omega\to\pi\gamma} &= -i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon^{\alpha}_{\omega}\epsilon^{\beta}_{\gamma} q^{\rho}k^{\sigma}\,\frac{4\,e}{M_V^2 M_\omega}\,\frac{F_V}{F}\Big[d_3\,M_\omega^2+(d_1+8d_2-d_3)m_\pi^2\Big]\,, \end{aligned} \tag{4.1}$$

where we have kept the generic mass M_V of the meson octet in the ρ propagator, in consistency with the procedure followed in the analysis of section 3; distinction is made between M_V and M_ω when the latter is of kinematic origin. Quite remarkably, if we now plug in the QCD constraints, eq. (3.10), obtained from the analysis of the short-distance behaviour of the $\langle$VVP$\rangle$ Green's function, we find a full prediction for this process:

$$i\,\mathcal{M}_{\omega\to\pi\gamma} = i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon^{\alpha}_{\omega}\epsilon^{\beta}_{\gamma} q^{\rho}k^{\sigma}\,\frac{e}{F_V}\left[\frac{N_C}{8\pi^2}\,\frac{M_\omega}{F}\; - \;\frac{F}{2}\,\frac{M_\omega}{M_V^2}\left(1+\frac{m_\pi^2}{M_\omega^2}\right)\right]. \tag{4.2}$$

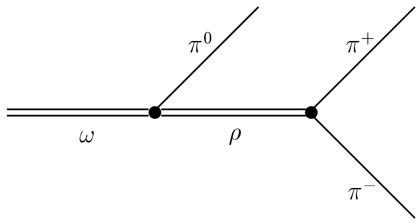
We notice that the direct (figure 2a) and the ρ exchange diagrams (figure 2b) almost contribute to similar extent to this process. This means that contrary to what we would expect from vector meson dominance, the $\omega\rho\pi$ coupling does not saturate the decay $\omega \to \pi\gamma$. The actual value of this coupling in our formalism,[5] d_3, is less than half of the one that would arise from VMD, where only the diagram figure 2b contributes. This has immediate consequences to other decay channels, as we shall see in the next subsection.

Finally, the width is easily obtained, giving

$$\Gamma(\omega\to\pi\gamma) = \frac{\alpha}{192}\,M_\omega\left(1-\frac{m_\pi^2}{M_\omega^2}\right)^3\left[\frac{N_C}{4\pi^2}\,\frac{M_\omega^2}{F^2}\; - \;\frac{M_\omega^2}{M_V^2}\left(1+\frac{m_\pi^2}{M_\omega^2}\right)\right]^2. \tag{4.3}$$

The relation $F_V = \sqrt{2}\,F$, consequence of conditions (3.1) and (3.2), has been employed in deriving the result in eq. (4.3). Varying the parameter F from the bare value $F_0 \simeq 87\,\mathrm{MeV}$ to the dressed one (i.e. the pion decay constant), $F_\pi \simeq 92.4\,\mathrm{MeV}$ [21], we get that our prediction for $\Gamma(\omega\to\pi\gamma)$ ranges from $0.703\,\mathrm{MeV}$ to $0.524\,\mathrm{MeV}$, with the choices $M_V = M_\rho = 771.1\,\mathrm{MeV}$ and $M_\omega = 782.6\,\mathrm{MeV}$ [21]. This 5–30% deviation from the experimental

[5]The tiny contribution coming from the pion mass contribution in $\mathcal{M}^{\rho}_{\omega\to\pi\gamma}$ can be obviated in this discussion.



$+ \qquad$ cyclic permutations of $\{\pi^0, \pi^+, \pi^-\}$

Figure 3: The $\omega \to \pi^+\pi^-\pi^0$ decay amplitude via an intermediate ρ exchange.

value, $\Gamma(\omega \to \pi\gamma)|_{\rm exp} = (0.734 \pm 0.035)\,\rm MeV$, is in accordance with the expected size of next-to-leading $1/N_C$ corrections. Also the $\rho \to \pi\gamma$ decays, related with $\omega \to \pi\gamma$ by a $\rm SU(3)_V$-symmetry factor, can be compared with the phenomenology. We obtain

$$\Gamma(\rho^\pm \to \pi^\pm\gamma) = \Gamma(\rho^0 \to \pi^0\gamma) = \frac{1}{9}\,\Gamma(\omega \to \pi\gamma) = 78.1 - 58.2 \text{ keV}\,,$$

depending on the value of the parameter F, to be compared with the experimental measurements, $\Gamma(\rho^\pm \to \pi^\pm\gamma)|_{\rm exp} = (67 \pm 7)\,\rm keV$ and $\Gamma(\rho^0 \to \pi^0\gamma)|_{\rm exp} = (120 \pm 30)\,\rm keV$. It is interesting to note that our predictions are rather close to the ones derived from the extended Nambu-Jona-Lasinio model [19].

Our result for $\Gamma(\omega \to \pi\gamma)$ is quite significant being a pure prediction of the matching procedure of the resonance effective theory with the OPE expansion given by QCD. The extension of our analysis to other decay channels (e.g. $K^* \to K\gamma$, $\phi \to \eta\gamma$) requires that exact $\rm SU(3)_V$ symmetry is left aside in order not to lose the predictive power shown in $\omega \to \pi\gamma$. This study would require to consider the OPE expansion in the asymptotic regime keeping distinct masses for each quark flavour, a rather non trivial task.

4.2 $\omega \to \pi^+\pi^-\pi^0$

The odd-intrinsic parity sector included in the resonance lagrangian can also account for the ρ-mediated mechanism of decay of the ω meson to the $\pi^+\pi^-\pi^0$ final state, figure 3. If we label as k_1, k_2, k_3 the momenta of the π^+, π^- and π^0 respectively, the amplitude associated to the diagram of figure 3, including cyclic permutations among k_1, k_2 and k_3, reads

$$\begin{aligned} i\,\mathcal{M}_{\omega\to 3\pi} = i\,\epsilon_{\alpha\beta\rho\sigma}\, k_1^\alpha k_2^\beta k_3^\rho \epsilon_\omega^\sigma \frac{8\,G_V}{M_\omega F^3} \Bigg[& \frac{m_\pi^2(d_1 + 8d_2 - d_3) + (M_\omega^2 + s_{12})\, d_3}{M_V^2 - s_{12}} + \\ & + \{s_{12} \to s_{13}\} + \{s_{12} \to s_{23}\} \Bigg]\,. \end{aligned} \tag{4.4}$$

The kinematic invariants are defined as usual, i.e. $s_{ij} = (k_i + k_j)^2$. The VMD hypothesis for this decay predicts that the amplitude above is the dominant one. Then the corresponding width would be calculated as

$$\begin{aligned} \Gamma(\omega \to \pi^+\pi^-\pi^0) = \frac{G_V^2}{4\,\pi^3\, M_\omega^5\, F^6} \int_{4m_\pi^2}^{(M_\omega - m_\pi)^2} ds_{13} \int_{s_{23}^{\rm min}}^{s_{23}^{\rm max}} ds_{23}\, \mathcal{P}(s_{13}, s_{23}) \times \\ \times \left[\frac{m_\pi^2(d_1 + 8d_2 - d_3) + (M_\omega^2 + s_{12})d_3}{M_V^2 - s_{12}} + \{s_{12} \to s_{13}\} + \{s_{12} \to s_{23}\} \right]^2 , \end{aligned} \tag{4.5}$$

where the function $\mathcal{P}$ is the polarization average of the tensor structure of $\mathcal{M}_{\omega\to 3\pi}$,

$$\mathcal{P}(s_{13}, s_{23}) = \frac{1}{12}\left\{-m_\pi^2(m_\pi^2 - M_\omega^2)^2 - s_{13}s_{23}^2 + (3m_\pi^2 + M_\omega^2 - s_{13})s_{13}s_{23}\right\}. \tag{4.6}$$

With $G_V = F/\sqrt{2}$ and the relations obtained by the short-distance matching, we find that the width above works out $\Gamma(\omega \to \pi^+\pi^-\pi^0) \simeq 1.4\,\mathrm{MeV}$, quite far from the experimental result [21], $\Gamma(\omega \to \pi^+\pi^-\pi^0)|_{\exp} = (7.52 \pm 0.06)\,\mathrm{MeV}$. Clearly, the contribution from a direct $\omega \to 3\pi$ amplitude must be larger than expected from VMD. Such deviation can be traced back to the result obtained in the previous section for $\omega \to \pi\gamma$. There we found that the d_3 parameter was less than half the value one should expect from a dominant role of the $\rho\omega\pi$ coupling. The $\omega \to 3\pi$ width calculated above, eq. (4.5), is essentially (neglecting the tiny piece driven by the pion mass squared) proportional to d_3^2; therefore, there is roughly a factor of ~ 4 between our calculation of $\Gamma(\omega \to \pi\rho \to 3\pi)$ and the result obtained under VMD by fixing the $\rho\omega\pi$ coupling from the $\omega \to \pi\gamma$ width (see for example ref. [22]). This factor would raise the result of (4.5) to $\sim 5.6\,\mathrm{MeV}$, i.e. reaching the level of accuracy of leading large-N_C calculations.

According to the precedent discussion, the intermediate meson exchange does not account entirely for the ω decay into three pions, and the direct terms must be considered.[6] In fact both contributions appear at the same order in the large-N_C expansion and the ρ resonance, being far off-shell in this process, does not resonate. Consequently, there is no reason that justifies neglecting the direct vertex. Indeed, it was pointed out in ref. [22] that VMD alone predicts a too large $\rho\omega\pi$ coupling with respect to what suggests naive chiral counting. The QCD-enforced appearance of a direct term in our approach, which has reduced the $\rho\omega\pi$ coupling to the half, casts some light on the issue.

4.3 $\pi \to \gamma\gamma$

In the chiral limit, the amplitude for the $\pi \to \gamma\gamma$ process is non-vanishing and exactly predicted by the ABJ anomaly [23], eq. (3.9). Away from this limit, the amplitude receives small contributions from different sources, including isospin-breaking effects, as well as electromagnetic and higher-order chiral corrections. As the loop contribution vanishes [24], the latter corrections start with the $\mathcal{O}(p^6)$ Goldstone chiral lagrangian. The odd-intrinsic-parity interactions among vector resonances introduced in section 2 also generate chiral corrections to this process proportional to m_π^2. Let us first study the numerical size of these corrections, fixed by virtue of the short-distance constraints.

The amplitudes for the decay via intermediate meson exchange, depicted in figure 4, give as a result

$$\begin{aligned} i\,\mathcal{M}^{(a)}_{\pi\to\gamma\gamma} &= -i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon_1^\alpha\epsilon_2^\beta k_1^\rho k_2^\sigma\;\frac{8\sqrt{2}}{3}\,\frac{e^2}{M_V}\,\frac{F_V}{F}\,\frac{m_\pi^2}{M_V^2}\left(c_1 + c_2 + 8c_3 - c_5\right) \\ &= 0\,, \end{aligned}$$

[6] In our effective theory, these terms would be obtained by writing down the operators which give rise to local contributions to $\omega \to 3\pi$.

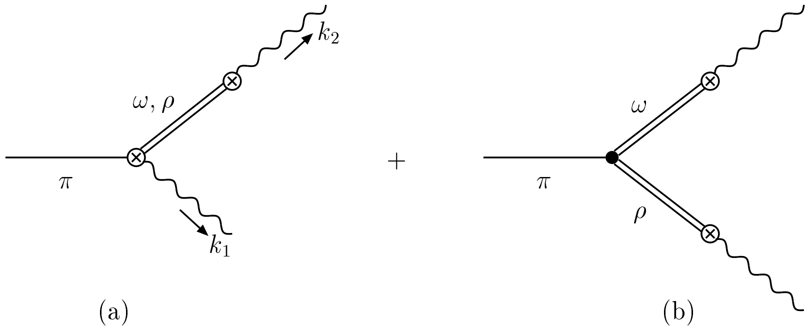


Figure 4: Feynman diagrams with vector mesons giving $\mathcal{O}(m_\pi^2)$ corrections to $\pi \to \gamma\gamma$ decay. The diagrams with interchanged photon momenta must be added.

$$\begin{aligned} i\,\mathcal{M}^{(b)}_{\pi\to\gamma\gamma} &= i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon_1^\alpha \epsilon_2^\beta k_1^\rho k_2^\sigma \frac{8\,e^2}{3\,F}\,\frac{F_V^2}{M_V^2}\,\frac{m_\pi^2}{M_V^2}\Big(d_1 + 8d_2 - d_3\Big) \\ &= i\,\epsilon_{\alpha\beta\rho\sigma}\,\epsilon_1^\alpha \epsilon_2^\beta k_1^\rho k_2^\sigma \,\frac{e^2\,F}{3M_V^2}\,\frac{m_\pi^2}{M_V^2}\,. \end{aligned} \tag{4.7}$$

The diagram with a VJP vertex vanishes after the short-distance conditions are applied, and the remaining contribution gets completely fixed. The correction induced into the $\pi \to \gamma\gamma$ width, by our result above gives:

$$\Gamma(\pi \to \gamma\gamma) = \frac{\alpha^2}{64\,\pi^3\,F^2} m_\pi^3\,[1-\Delta]^2\,, \tag{4.8}$$

where

$$\Delta = \frac{4\pi^2}{3}\,\frac{F^2}{M_V^2}\,\frac{m_\pi^2}{M_V^2} \;\simeq\; 0.006\,. \tag{4.9}$$

This result provides a tiny 1% correction to the width, and it is perfectly compatible with the experimental uncertainty, $\Gamma(\pi\to\gamma\gamma)|_{\rm exp} = (7.7 \pm 0.6)\,{\rm eV}$.

This evaluation of the amplitude for the $\pi \to \gamma\gamma$ process could also have been carried out within the chiral lagrangian $\mathcal{L}^{(6)}_{\rm odd}$ of eq. (2.13), where only the operator with t_1 contributes.[7] With $t_1 \simeq t_1^V$ and using the value given in eq. (3.12) we obtain the result above. The exercise carried out in this Subsection, evaluating the diagrams in figure 4, shows explicitly that only the two-resonance driven amplitude gives contribution to this process in the antisymmetric formulation.

5. Conclusions

Effective theories of QCD carry all-important features of the underlying theory to describe the relevant hadron dynamics in the non-perturbative regime. The odd-intrinsic-parity sector has been studied within chiral perturbation theory but its extension to the energy

[7]The operator with the t_2 coupling only contributes if one of the photons is off-shell.

region of the resonances requires a proper implementation of the active degrees of freedom and to generate the effective theory through a procedure able to enforce the relevant dynamics on the coupling constants. This task has been addressed in this paper. After considering the operators of the lagrangian, that rely on the global symmetries of QCD, we proceed to drive the information, from the underlying theory onto the couplings, through a matching with the leading OPE of the Green's function in the chiral limit.

Let us highlight the main results that can be extracted from the previous sections.

- The lowest order lagrangian involving interactions among one Goldstone mode and two vector particles has been introduced in the Resonance Chiral Theory with the vector resonances described in terms of antisymmetric tensor fields.

- The vector-vector-pseudoscalar three-point-function $\langle$VVP$\rangle$ has been calculated at tree level with the new sector added to the resonance lagrangian. Assuming that a matching procedure between the result obtained from the effective action and from QCD in terms of massless quarks is reliable at large momenta, we have derived a set of relations among the parameters of the odd-intrinsic-parity sector.

- In contrast to the result of ref. [7], where vector resonances were described in the Proca formalism, the expression for the $\langle$VVP$\rangle$ Green's function obtained from the lagrangian with antisymmetric tensor fields is fully compatible with the short-distance QCD constraints, which reduce it to the ansatz suggested by LMD in the large-N_C limit of QCD, successfully tested in previous works [9, 11].

- On the way, we have found that the same combinations of couplings which appears in the short-distance QCD constraints, show up in the $\omega \to \pi\gamma$ and $\rho \to \pi\gamma$ amplitudes calculated with the resonance lagrangian, thus allowing us to give a full prediction for these decays. The agreement with the experimental values is remarkable.

- The $\omega \to \pi\gamma$ calculation above shows an important feature: the contribution from a direct $\omega\pi\gamma$ vertex is larger than expected from VMD. Indeed, it amounts to more than 50% of the total result for this amplitude. This agrees with the expectations from the $1/N_C$ counting, as both mechanisms contribute to the same order.

- The last point has an important consequence for other channels where VMD alone was thought to be the relevant mechanism of decay. To serve as an example, we have shown that the intermediate meson exchange $\omega \to \rho\pi \to 3\pi$ cannot dominate the $\omega \to 3\pi$ process in our framework, and the local contribution thus becomes essential.

Our study has shown that the use of effective theories of QCD in the intermediate energy region, populated by resonances, endows the basic information to provide both qualitative and quantitative descriptions of the hadron phenomenology in a model-independent way. Consequently it provides a compelling framework to work with.

Acknowledgments

We wish to thank V. Cirigliano for his helpful comments and for reading the manuscript. The work of P. D. Ruiz-Femenía has been partially supported by a FPU scholarship of the Spanish *Ministerio de Educación y Cultura*. J. Portolés is supported by a "Ramón y Cajal" contract with CSIC funded by MCYT. This work has been supported in part by TMR EURIDICE, EC Contract No. HPRN-CT-2002-00311, by MCYT (Spain) under grant FPA2001-3031, and by ERDF funds from the European Commission.

A. Construction of the odd-intrinsic-parity resonance lagrangian

Within the antisymmetric formulation, the integration of a vector meson gives a contribution which starts at $\mathcal{O}(p^4)$ in the chiral counting. Interaction terms with a Levi-Civita tensor start to contribute[8] at $\mathcal{O}(p^6)$, as terms with one vector meson and an $\mathcal{O}(p^2)$ chiral tensor are not charge conjugation or parity invariant, and a possible term with two resonance fields, $\epsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}V^{\rho\sigma}\rangle$, is forbidden by parity conservation. Besides, terms of odd order, i.e. $\mathcal{O}(p^3)$ or $\mathcal{O}(p^5)$, cannot be written down in the presence of an $\epsilon_{\mu\nu\rho\sigma}$ tensor. Either a chiral tensor of $\mathcal{O}(p^4)$ together with a vector meson is needed, giving rise to the VJP terms, or two vector resonances and a chiral tensor of $\mathcal{O}(p^2)$ (VVP terms).

The available chiral tensors have already been introduced in section 2: $\chi_\pm$, $f_\pm^{\mu\nu}$ are $\mathcal{O}(p^2)$, while the covariant derivative ∇_α and u_α count as $\mathcal{O}(p)$. These tensors have defined transformation properties under chiral rotations and thus allow us to write down chiral invariant objects in a straightforward way.

Let us first give some clues about the construction of the VVP basis. Aside from the two vector mesons, we should consider all possible tensors giving one pseudoscalar. Therefore, we can have:

- One covariant derivative ∇_μ and one u_ν tensor, with the covariant derivative acting on either the resonance fields or the pseudoscalar u_ν. In the latter case $\nabla_\mu u_\nu$ is symmetric in its indices for the linear term of the expansion of u_ν in terms of Goldstone fields:
$$u_\nu = -\frac{\sqrt{2}}{F}\,\partial_\nu\Phi + \text{terms with 3 pseudoscalar fields} + \cdots\,.$$
- A χ_- external field, whose expansion in terms of the pseudoscalar octet of fields starts with one particle states. A χ_+ external field together with the two vector mesons is however not allowed by parity conservation.

In addition, the Schouten identity,

$$g_{\rho\sigma}\epsilon_{\alpha\beta\mu\nu} + g_{\rho\alpha}\epsilon_{\beta\mu\nu\sigma} + g_{\rho\beta}\epsilon_{\mu\nu\sigma\alpha} + g_{\rho\mu}\epsilon_{\nu\sigma\alpha\beta} + g_{\rho\nu}\epsilon_{\sigma\alpha\beta\mu} = 0\,, \tag{A.1}$$

reduces the number of independent operators because it may establish relations among those with different ordering of the Lorentz indices. As an example, consider the two

[8] For terms involving vector resonances, this counting should be understood as the one obtained after integrating out the resonances, i.e. the order of the chiral operator induced by vector exchange.

following VVP terms:

$$\begin{aligned}\mathcal{O}^1 &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu},V^{\rho\alpha}\}\nabla_\alpha u^\sigma\,\rangle = g_{\alpha\lambda}\,\epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu},V^{\rho\lambda}\}\nabla^\alpha u^\sigma\,\rangle\,,\\ \mathcal{O}^2 &= \epsilon_{\mu\nu\rho\sigma}\,\langle\,\{V^{\mu\nu},V^{\rho\sigma}\}\nabla_\alpha u^\alpha\,\rangle = g_{\alpha\sigma}\,\epsilon_{\mu\nu\rho\lambda}\,\langle\,\{V^{\mu\nu},V^{\rho\lambda}\}\nabla^\alpha u^\sigma\,\rangle\,.\end{aligned}$$

With the identity (A.1) we find that the second operator is proportional to the first one:

$$\mathcal{O}^2 = 4\,\mathcal{O}^1\,.$$

Similarly, the Schouten identity must be applied to operators with the ∇_μ acting on the resonance fields and to operators from the VJP sector to further reduce the basis.

To close with the analysis of the VVP interactions, recall that a term $\sim\langle V^{\mu\nu}V^{\rho\sigma}f_-^{\alpha\beta}\rangle$ would include an external vector (or axial-vector) source in addition to the wanted pseudoscalar. Clearly, these terms do not belong to our VVP sector.

For the VJP interactions, basically the same considerations made above hold, and the substitution of one of the resonance fields $V^{\mu\nu}$ by and external vector field $f_+^{\mu\nu}$, which has the same properties under P and C transformations, gives the allowed VJP structures. Note that for each VVP term two VJP operators emerge with this procedure (except for the term with χ_-), as the vector tensors are not equal now. We have chosen that ∇^α acts on the vector meson or on the pseudoscalar field to define the final set of independent VJP operators. As quoted in the main text, the term $\mathcal{O}^4_{\mathrm{VJP}}$, where the pseudoscalar now comes up from the $f_+^{\mu\nu}$ tensor, is a $\mathrm{SU}(3)_{\mathrm{V}}$-breaking operator. Indeed its lower order expansion in terms of Goldstone fields is proportional to $m_K^2 - m_\pi^2$.

References

[1] S. Weinberg, *Phenomenological lagrangians*, *Physica* **96A** (1979) 327.

[2] J. Gasser and H. Leutwyler, *Chiral perturbation theory to one loop*, *Ann. Phys. (NY)* **158** (1984) 142.

[3] J. Gasser and H. Leutwyler, *Chiral perturbation theory: expansions in the mass of the strange quark*, *Nucl. Phys.* **B 250** (1985) 465.

[4] S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological lagrangians, 1*, *Phys. Rev.* **177** (1969) 2239;
C.G. Callan, S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological lagrangians, 2*, *Phys. Rev.* **177** (1969) 2247.

[5] G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The role of resonances in chiral perturbation theory*, *Nucl. Phys.* **B 321** (1989) 311.

[6] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral lagrangians for massive spin 1 fields*, *Phys. Lett.* **B 223** (1989) 425.

[7] M. Knecht and A. Nyffeler, *Resonance estimates of $\mathcal{O}(p^6)$ low-energy constants and QCD short-distance constraints*, *Eur. Phys. J.* **C 21** (2001) 659 [hep-ph/0106034].

[8] J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, *QCD short-distance constraints and hadronic approximations*, *J. High Energy Phys.* **04** (2003) 055 [hep-ph/0304222].

JHEP07(2003)003

[9] B. Moussallam, *Chiral sum rules for parameters of the order six lagrangian in the W-Z sector and application to π^0, η, $\eta\prime$ decays*, *Phys. Rev.* **D 51** (1995) 4939 [hep-ph/9407402].

[10] B. Moussallam, *A sum rule approach to the violation of dashen's theorem*, *Nucl. Phys.* **B 504** (1997) 381 [hep-ph/9701400].

[11] M. Knecht, S. Peris, M. Perrottet and E. de Rafael, *Decay of pseudoscalars into lepton pairs and large-N_c QCD*, *Phys. Rev. Lett.* **83** (1999) 5230 [hep-ph/9908283].

[12] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.

[13] E. Witten, *Baryons in the 1/N expansion*, *Nucl. Phys.* **B 160** (1979) 57.

[14] J. Wess and B. Zumino, *Consequences of anomalous Ward identities*, *Phys. Lett.* **B 37** (1971) 95;
E. Witten, *Current algebra, baryons, and quark confinement*, *Nucl. Phys.* **B 223** (1983) 22.

[15] D. Issler, *Nonrenormalization of the chiral anomaly in chiral perturbation theory*, SLAC-PUB-4943;
R. Akhoury and A. Alfakih, *Invariant background field method for chiral lagrangians including Wess-Zumino terms*, *Ann. Phys. (NY)* **210** (1991) 81.

[16] J. Bijnens, L. Girlanda and P. Talavera, *The anomalous chiral lagrangian of order p^6*, *Eur. Phys. J.* **C 23** (2002) 539 [hep-ph/0110400].

[17] J. Bijnens, G. Ecker and J. Gasser, *Chiral perturbation theory*, hep-ph/9411232.

[18] E. Pallante and R. Petronzio, *Anomalous effective lagrangians and vector resonance models*, *Nucl. Phys.* **B 396** (1993) 205.

[19] J. Prades, *Massive spin 1 field chiral lagrangian from an extended nambu-jona-lasinio model of QCD*, *Z. Physik* **C 63** (1994) 491 [hep-ph/9302246], erratum *ibid.* **C11** (1999) 571.

[20] J. Bijnens, *Chiral perturbation theory and anomalous processes*, *Int. J. Mod. Phys.* **A 8** (1993) 3045.

[21] Particle Data Group collaboration, K. Hagiwara et al., *Review of particle physics*, *Phys. Rev.* **D 66** (2002) 010001.

[22] G. Ecker and R. Unterdorfer, *Four-pion production in e^+e^- annihilation*, *Eur. Phys. J.* **C 24** (2002) 535 [hep-ph/0203075].

[23] S.L. Adler, *Axial vector vertex in spinor electrodynamics*, *Phys. Rev.* **177** (1969) 2426;
J.S. Bell and R. Jackiw, *A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the sigma model*, *Nuovo Cim.* **A60** (1969) 47;
W.A. Bardeen, *Anomalous Ward identities in spinor field theories*, *Phys. Rev.* **184** (1969) 1848.

[24] J.F. Donoghue, B.R. Holstein and Y.C.R. Lin, *Chiral loops in $\pi^0, \eta^0 \to \gamma\gamma$ and $\eta\eta'$ mixing*, *Phys. Rev. Lett.* **55** (1985) 2766.

JHEP07(2003)003